FOUR-DIMENSIONAL JETS AS UNIVERSAL CHARACTERISTICS OF MULTIPLE PARTICLE PRODUCTION

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A new definition of the jets as clusters in the
four-velocity space has been used to make an inva-
riant analysis of multiple particle production pro-
cesses in pp , $\bar{p}p$, $\pi^- p$, $\pi^- C$, pC and pTa collisions
at energies from 6 to 205 GeV. It has been observed
that the characteristics of four-dimensional jets
are universal, i.e., independent of neither the pro-
perties, nor the energy of colliding particles for
$P \geq 22$ GeV/c.

The investigation has been performed at the La-
boratory of High Energies, JINR.

Четырехмерные струи - универсальные характеристики
множественного рождения частиц

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На основе нового определения струй как кластеров
в четырехмерном пространстве скоростей проведен ин-
вариантный анализ множественных процессов при столк-
новении $\bar{p}p$, $\pi^- p$, $\pi^- C$, pTa , pC и pp в области энер-
гий от 6 до 205 ГэВ. Показано, что характеристики
четырехмерных струй универсальны, т.е. не зависят
ни от свойств сталкивающихся частиц, ни от энергии
столкновения для $P_{лаб} \geq 22$ ГэВ/c.

Работа выполнена в Лаборатории высоких энергий
ОИЯИ.

The traditional analysis of the jet behaviour of se-
condary particles with the aid of variables "spherici-
ty", "trust" and others deals with relativistic noninva-
riant quantities and the jet characteristics depend not
only on the frame of reference, but also on the proper-
ties of colliding particles. A new method of describing
multiple particle production processes, which employs
only Lorentz invariant quantities, has been suggested
in papers[1] devoted to the study of relativistic nuc-
lear collisions.

The aim of the present paper is to report results of the analysis of jet production by the new method in various processes ($\bar{p}p$, $\pi^- p$, $\pi^- C$, pC, pTa and pp) in a wide energy range from 6 to 205 GeV. The main result of this analysis is the discovery of the fact that the four-dimensional jets are universal.

In the invariant method of analysing multiple particle production, the processes

$$I + II \rightarrow 1 + 2 + 3 + \ldots \tag{1}$$

are considered in a space the points of which are the four-velocities $u_i = P_i/m_i$ or the four-moments P_i of the particles divided by their masses m_i. The positive invariant quantities having the meaning of the squared distances in this space

$$b_{ik} = -(u_i - u_k)^2 = 2[(u_i u_k) - 1], \tag{2}$$

where $i,k = I, II, 1, 2, 3, \ldots$, are basic variables describing the relative particle motion.

The idea of the introduction of the variables b_{ik} consists in that in the cross sections of the processes (1) the following statistical regularity is realized: in definite domains of these variables (see below) the b_{ik} distributions monotonously and rather rapidly decrease with increasing b_{ik}. This fact reflects the quark fundamental property – asymptotic freedom, that is, vanishing of interactions at small distances or at $b_{ik} \rightarrow \infty$. We divide the whole set of particles into groups: the first group comprises m_α particles; the second one, m_β particles, and consider the case when $b_{\alpha\beta} = -(V_\alpha - V_\beta)^2 \rightarrow \infty$, here V_α is the middle point in the group α; V_β, the one in the group β. In other words, $(u_i^\alpha V_\alpha) - (u_j^\beta V_\beta) \sim 1$, but $(u_i^\alpha V_\beta) - (u_j^\beta V_\alpha) \sim (V_\alpha V_\beta) \rightarrow \infty$. The set of m_α particles is described in terms of the variables $\{b_{\alpha i}, (u_i^\alpha N_\beta), \phi_i\}$ and set of m_β particles in terms of the variables $\{b_{\beta j}, (u_j^\beta N_\alpha)\phi_j\}$. Here

$$N_\alpha = \frac{V_\alpha}{(V_\alpha \cdot V_\beta)} \quad \text{and} \quad N_\beta = \frac{V_\beta}{(V_\alpha \cdot V_\beta)} \tag{3}$$

ϕ_i and ϕ_j are azimuthal angles, $(u_i^\alpha \cdot N_\beta)$ and $(u_j^\beta \cdot N_\alpha)$ at $(V_\alpha \cdot V_\beta) \rightarrow \infty$ transform to the light cone variables $x_i^\alpha = u_{i0}^\alpha - u_{iz}^\alpha$ and $x_j^\beta = u_{j0}^\beta - u_{jz}^\beta$, respectively.

Using this notation, it is possible to formulate the correlation depletion principle as a general property of the invariant distributions describing multiple particle

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production

$$F \{ ..., b_{\alpha i}, (u_i^{\alpha} \cdot N_{\beta}), ...; b_{\alpha\beta}, ... b_{\beta j}, (u_j^{\beta} \cdot N_{\alpha}), ... \} | \rightarrow$$
$$b_{\alpha\beta} \rightarrow \infty \quad (4)$$
$$\rightarrow F^{\alpha} \{ ..., b_{\alpha i}, x_i^{\alpha}, ... \} \cdot F^{\beta} \{ ..., b_{\beta j}, x_j^{\beta} ... \}.$$

This principle is analogous to the correlation depletion principle suggested by Bogolubov in statistical physics. According to our concepts, the jets are a particular case of the factorization (4). We determine the jet axis as a single four-vector V that is extracted from the condition of minimum of the quantity

$$\sum_k b_k = - \sum_k (V - u_k)^2 . \quad (5)$$

The summation is performed over the particles belonging to a separated group of particles. The quantity (5) is minimal for

$$V = \frac{\sum\limits_k u_k}{\sqrt{(\sum\limits_k u_k)^2}} . \quad (6)$$

As investigations in the field of relativistic nuclear physics show, the transition of hadron interactions to the quark-gluon level sets is rather early[2]: for

$$b_{ik} \geq 5. \quad (7)$$

Hence, the jets were expected to be separated already for $(u_I \cdot V) \sim (u_{II} \cdot V) \sim 3,5$ and the width of the four-dimensional jets to be equal to about the same value.

The study of jet production in soft hadron collisions has shown that in these processes it is observed the production of two jets, emitted in the forward and backward hemispheres in the center-of-mass system, the characteristics of which depend on the type of a fragmenting system (quark or diquark) and the collision energy[3,4]. However, as is shown below, the difference in the jet characteristics is due to a relativistic noninvariant approach. In the present paper the selection of the particles belonging to the jets is made by means of relativis-

tic invariant variables [*]

$$x_p^i = \frac{m_i}{m_I}(u_i \cdot N_{II}) \geq (0.1 \div 0.2), \text{ where } N_{II} = \frac{u_{II}}{(u_I \cdot u_{II})} \quad (8)$$

or

$$x_t^i = \frac{m_i}{m_{II}}(u_i \cdot N_I) \geq (0.1 \div 0.2), \text{ where } N_I = \frac{u_I}{(u_I \cdot u_{II})} \; .$$

For the case of nuclear collisions the atomic mass unit $m_0 = 931$ MeV is substituted for m_{II}. As the criterion of the particle selection with respect to x_p and x_t becomes more strict, we select such hadron jets that carry away an ever-growing fraction of the four-momentum of primary hadrons. The hadrons that cannot be attributed to one or another jet are in the range $x_p \geq 0.2$ and $x_t \geq 0.2$. They make up less than 1% of the secondary particles and their origin is due to hard processes. In the range $x_p \geq 0.1$ and $x_t \geq 0.1$ the fraction of such hadrons increases up to 2%. Within the accuracy of the experimental data in question, good characteristics of the fragmentation region for the projectile are $x_p \geq 0.1$ and $x_t \leq 0.1$, and for the target are $x_t \geq 0.1$ and $x_p \leq 0.1$.

The present study was performed on the basis of a set of experimental data on hadron-hadron and hadron-nucleus collisions of various types in an energy range from 6 to 205 GeV. Approximately 220 thousand events were analysed. The data on 40 GeV/c $\pi^- p$ and $\pi^- C$ collisions and 22.4 GeV/c $\bar{p}p$ collisions were obtained with the aid of a 200 cm propane and a 200 cm hydrogen ("Liudmila") bubble chambers irradiated by π^- and $\bar{p}$ beams at the Serpukhov accelerator. The data on 10 GeV $p(C_3H_8)$ and pTa collisions were obtained by irradiating the 200 cm propane bubble chamber with Ta plates inside its working volume at the Synchro-phasotron. The data on 205 GeV/c pp collisions and 5.7 and 12 GeV/c $\bar{p}p$ collisions were obtained with the aid of a 76 cm hydrogen chamber (FNAL) and a 81 cm and a 200 cm hydrogen chambers (CERN). The experimental details are described in refs.[5-11]. Data summary tapes containing information about the kinematical parameters of events had been used.

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[*] A more general method of jet separation is to find correlated particle groups in the b_{ik} space[1] with correlation radius $\langle b_{ik} \rangle \sim 1$. This is especially important for the separation of multi-jet events.
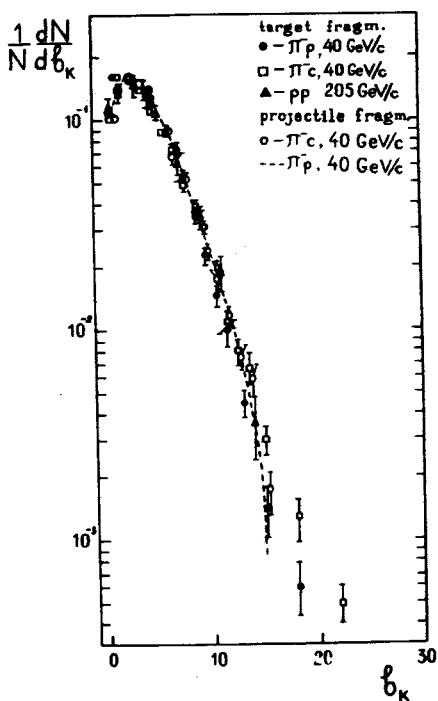
$$\frac{1}{N}\frac{dN}{db_K}$$

target fragm.
- ● − $\pi^- p$, 40 GeV/c
- □ − $\pi^- c$, 40 GeV/c
- ▲ − pp 205 GeV/c

projectile fragm.
- ○ − $\pi^- c$, 40 GeV/c
- --- $\pi^- p$, 40 GeV/c

Fig.1. The b_k distribution of π^- mesons in jets for the fragmentation of different projectiles ($x_p \geq 0.1$, $x_t \leq 0.1$) and targets ($x_t \geq 0.1$, $x_p \leq 0.1$).
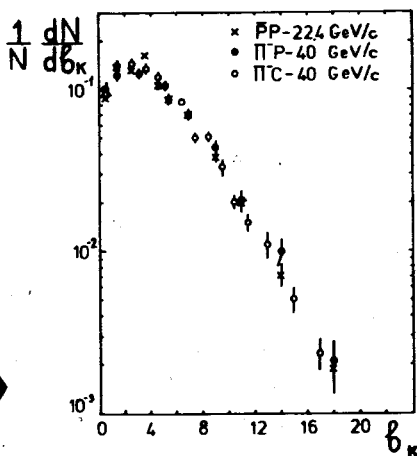
$$\frac{1}{N}\frac{dN}{db_K}$$

- × $\bar{p}p$−22.4 GeV/c
- ● $\pi^- p$−40 GeV/c
- ○ $\pi^- c$−40 GeV/c

Fig.2. The b_k distribution of π^- mesons in jets for the fragmentation of different targets ($x_t \geq 0.2$, $x_p \leq 0.2$).

Figure 1 gives the b_k distributions of π^- mesons for 40 GeV/c $\pi^- p$ and $\pi^- C$ collisions and 205 GeV/c pp collisions in the region of fragmentation of targets (p, C) for $x_t \geq 0.1$ and $x_p \leq 0.1$ and projectiles (π^-,p) for $x_p \geq 0.1$ and $x_t < 0.1$. In figure 2 we give the same distributions for 22.4 GeV/c $\bar{p}p$ collisions and for 40 GeV/c $\pi^- p$ and $\pi^- C$ collisions in the target fragmentation region at $x_t \geq 0.2$ and $x_p \leq 0.2$.

In order to obtain the jet characteristics under identical experimental conditions in various processes identified protons with $P < 0.8$ GeV/c were excluded from the consideration, and the ones with $P > 0.8$ GeV/c were taken to be π^+ mesons. In the target fragmentation region, because of an admixture of nonidentified protons which affect the obtaining of the jet axis, the $\langle b_k \rangle$ values for π^- mesons can be overestimated by about 10%.

It can be seen from the figures that in the considered energy range 22÷205 GeV/c the obtained distributions are

Table 1

$<b_k>$ values for π^--mesons

P_{lab}	Fragmentation region	Type of collision	$<b_k>$
205 GeV/c	$x_t \geq 0.2$	pp	5.0+0.4
	$x_p \leq 0.2$		
40 GeV/c	$x_p \geq 0.2$	$\pi^- p$	4.7+0.1
	$x_t \leq 0.2$	$\pi^- C$	4.8+0.1
40 GeV/c	$x_t \geq 0.2$	$\pi^- p$	5.0+0.1
		$\pi^- C$	5.0+0.1
	$x_p \leq 0.2$	$\pi^- C$ (x > 1)	5.7+0.1
22.4 GeV/c		$\bar{p}p$	4.66+0.08
12 GeV/c	$x_t \geq 0.2$	$\bar{p}p$	4.08+0.04
10 GeV/c	$x_p \leq 0.2$	$p(C_3H_8)+pTa$	2.8+0.1
5.7 GeV/c		$\bar{p}p$	3.67+0.02

identical and independent of neither the type of a fragmenting hadron (π^-, p, $\bar{p}$, C), nor the energy. Table 1 gives the average $<b_k(\pi^-)>$ values for $\bar{p}p$, $\pi^- p$ and $\pi^- C$ collisions in an energy range 5,7-205 GeV/c. Also they are identical for $P \geq 22.4$ GeV/c.

These results imply that the fragmentation of quarks and diquarks in the b_{ik} variables has a universal character and is independent of the energy in contrast to the traditional noninvariant approach. From the same data it follows that the carbon nucleus does not affect the jet formation, which gives evidence of the fact that the quark hadronization proceeds outside the nucleus.

A noticeable decrease of the $<b_k(\pi^-)>$ values at lower energies (5.7 and 12 GeV) is likely due to a considerable influence of the phase volume boundary and an important contribution of $\bar{p}p$ annihilation processes that have a multijet character.

Of a special interest is the study of cumulative jets, that is, the jets produced out of quarks which are knocked out from the multiquark configurations arising

in the nuclei. Such events are formally given by

$$X = \sum_i x_i = \frac{1}{m_0} \sum_i (P_i \cdot N_I) \geq 1. \tag{9}$$

The summation is performed over all the pions entering the jet ($x_i^t \geq 0.2, x_i^p \leq 0.2$). From the data obtained in such a way (Table 1) it follows that the characteristics of the cumulative jets are close to the ones which are due to the fragmentation of ordinary hadrons.

A similar analysis of the b_k distributions was also made for the strange K_s^0 and Λ particles from collisions of various types. The average $\langle b_k(K_s^0) \rangle$ and $\langle b_k(\Lambda) \rangle$ values for $\pi^- p$, $\pi^- C$ and $\bar{p}p$ collisions at $P = 40$ and 22.4 GeV/c are presented in Table 2. In the considered energy range the $\langle b_k \rangle$ values for K_s^0 and Λ particles, in just the same way as for pions, are seen to be independent of neither the type of a fragmenting system ($\bar{p}$, p, π, C), nor the primary collision energy. However, it should be emphasized that the average $\langle b_k \rangle$ values for strange particles are smaller than the ones for pions by about a factor of $3 \div 4$.

In conclusion, we have studied the single-particle distributions of hadrons ($\pi^\pm$, Λ, K_s^0) in jets in the four-velocity space. A total of 220 thousand events obtained with the aid of various detectors and accelerators in the incident particle momentum range from 6 to 205 GeV/c has been used to show that the four-dimensional jet characteristics have a universal character. They are independent of neuther the type of fragmenting particles (π^+, p, $\bar{p}$ nuclei), nor the primary hadron momentum (for $P \geq 22$ GeV/c). The difference between the properties of the universal four-dimensional jets and the jets obtained by traditional methods has been found to be essential. The independence of the jet characteristics of the fragmenting system properties (especially for nuclear collisions) gives evidence that the b_k distributions are due to the vacuum properties, the QCD particular features at large distances. The QCD interpretation of these distributions can give new universal parameters of strong interaction theory.

Table 2

$\langle b_k \rangle$ values for K_s^0 mesons

P_{lab}	Fragmentation region	Type of collision	$\langle b_k \rangle$
	$x_p \geq 0.1$	$\pi^- p$	1.74+0.07
40 GeV/c	$x_t \leq 0.1$	$\pi^- C$	1.7+0.1
	$x_t \geq 0.1$	$\pi^- p$	1.48+0.08
	$x_p \leq 0.1$	$\pi^- C$	1.6+0.1
22.4 GeV/c	$x_t \geq 0.2$ $x_p \leq 0.2$	$\bar{p}p$	1.52+0.06

$\langle b_k \rangle$ values for Λ particles

P_{lab}	Fragmentation region	Type of collision	$\langle b_k \rangle$
	$x_p \geq 0.1$	$\pi^- p$	1.8+0.1
40 GeV/c	$x_t \leq 0.1$	$\pi^- C$	1.5+0.2
	$x_t \geq 0.1$	$\pi^- p$	1.15+0.07
	$x_p \leq 0.1$	$\pi^- C$	1.21+0.08
22.4 GeV/c	$x_t \geq 0.2$ $x_p \leq 0.2$	$\bar{p}p$	0.94+0.05

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